Research question

This is a mathematical problem from space orbital mechanics. The following equation

\[
\Delta v_{\text{bi}} = \left(\frac{2}{\sqrt{1+xy}} - 1\right) + \sqrt{y} \left(\frac{2}{\sqrt{1+x}} - 1\right) + \sqrt{2xy} \left(\frac{1}{\sqrt{1+1/x}} - \sqrt{1+1/(xy)}\right) - (1-\sqrt{y}) \left(\frac{1+\sqrt{y}}{\sqrt{(1+y)/2}} - 1\right)
\]

is the normalized difference between the transition effort of a so-called bi-elliptic transfer between an inner and an outer circular orbit with radius \( a \) and \( a_\perp \), respectively, via an interim orbit with radius \( r_\perp \) (first three terms) and a direct Hohmann transfer (last term). Here

\[
y = a/a_\perp \leq 1 \\
x = a_\perp / r_\perp \leq 1
\]

A contour plot of the function delivers the shown figure, where the bright part indicates where the bi-elliptic transfer is more efficient. Of practical importance is the borderline to the dark area where \( \Delta v_{\text{bi}} = 0 \).

As can be verified by insertion, for \( x = 1 \), \( \Delta v_{\text{bi}} = 0 \) delivers \( y = 0 \). However, it turns out that for the limit \( x \to 1 \) the end-border-point is \( y \approx 0.0641778 \) and hard to determine more precisely. So, the problem statement reads:

For the limit \( x \to 1 \), what is the \( y \) to at least 10 decimal places that satisfies

\[
\Delta v_{\text{bi}} (x \to 1, y) = 0
\]

Possibly show that no such limit \( y > 0 \) exists.